Assignment 7

Coverage: 16.1 in Text.

Exercises: 16.1 no 10, 12, 15, 21, 22, 25, 27, 30, 32.

Hand in 16.1 no 15, 25, 30 by March 16.

Supplementary Problems

1. Find a parametric curve $\gamma(t),\ t \in [0,1]$, which describes the triangle with vertices at $(0,0),(2,0)$ and $(2,5)$ in anticlockwise direction.

2. Find the arc-length parametrization of the line segment $y = ax + b,\ x \in [0,2]$.

3. Show that the perimeter (length) of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ , \quad 0 < a < b \ ,$$

   is given by

$$\frac{4}{b} \int_0^1 \sqrt{\frac{1 - m^2t^2}{1 - t^2}}\, dt \ , \quad m^2 = 1 - \frac{a^2}{b^2} \ .$$

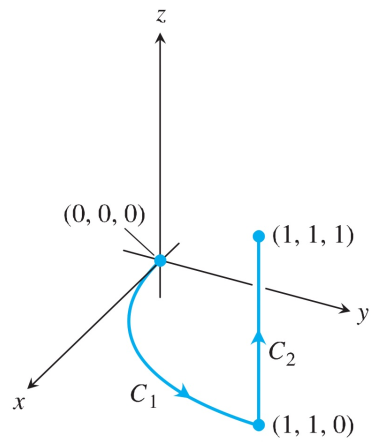
4. Optional. We always consider C^1- or even regular parametric curves in this chapter. It is clear from the definition of line integral why C^1 is necessary. However, to define a parametric curve only continuity is required. Look up "Peano curve" or "spacing filling curves" in Wiki to understand how wild a continuous parametric curve could be.

Evaluating Line Integrals over Space Curves

15. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path from (0, 0 to (1, 1, 1) (see accompanying figure) given by

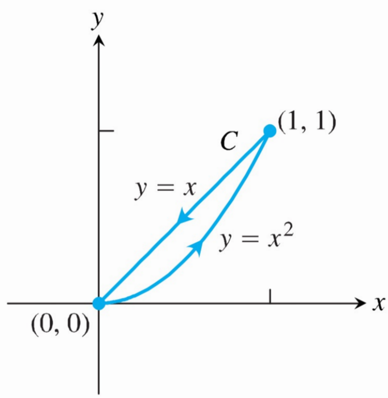
$$C_1: \quad \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le 1$$

$$C_2: \quad \mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$$



Line Integrals over Plane Curves

25. Evaluate $\int_C \left(x + \sqrt{y}\right) ds$ where C is given in the accompanying figure.



In Exercises 27–30, integrate f over the given curve.

30. $f(x, y) = x^2 - y, \quad C: \quad x^2 + y^2 = 4$ in the first quadrant from $(0, 2)$ to $(\sqrt{2}, \sqrt{2})$